Survival Estimation: Known-Fate Models

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Some Known-fate Models

- **Binomial survival model**
- **Nest success**
- **Radiotelemetry data**
- **Study design**

Binomial Survival Model

- Follow n subjects, x of them survive with probability s

$$f(x \mid n,s) = \binom{n}{x} s^{x} (1-s)^{n-x}$$

$$\hat{s} = \frac{x}{n}; \hat{\text{var}}(\hat{s}) = \frac{\hat{s}(1-\hat{s})}{n}$$

- Independent
- All detected
- No censoring

Mule Deer Example

	Number Released	Alive	Dead	Other
Treatment	61	19	38	4
Control	59	21	38	0

Treatment group:

$$\hat{s} = \frac{19}{57} == .333; \hat{\text{var}}(\hat{s}) = \frac{0.333(1-0.333)}{57} = 0.003899$$

Nest Success

- Important component of reproductive rate for may species (e.g., birds, many reptiles)
- Definition:
  - Pr (new nest succeeds to produce at least one hatchling or fledgling)
- Used with mean hatchlings/fledglings per successful nest

Inference About Nest Success: Example

- Random sample of n bird nests
  - y = 1 represents nest success, y = 0 represents nest failure
  - 10 nests are observed daily until success or failure
  - outcome $\underline{y}$ = {1, 0, 1, 1, 1, 0, 0, 0, 1, 1}
- Assume: nest fates are independent and identically distributed with unknown probability p of nest success. We can model the probability associated with success
- Pr(x successes out of 10 nesting attempts) is binomial:

$$f(x \mid p) = \binom{10}{x} p^{x} (1-p)^{10-x} \qquad L(p \mid x=6) = \binom{10}{6} p^{6} (1-p)^{4}$$

$$\frac{6}{p} - \frac{4}{1-p} = 0, \qquad \hat{p} = 0.6.$$

Is this OK?

Nest Studies and the Mayfield Method

- Nest searches estimate nest success
  - Many nests encountered late in nesting phase
  - Positive bias in survival
    - "Early" nests have more survival days
    - Chance of failure related to # of days remaining until success
- Harold Mayfield (amateur birder) proposed idea
  - Estimate a daily nest survival probability, *s*, from sample
  - Use knowledge of number of days in entire nesting cycle, *d*, to estimate nesting success, *S*, as:

$$\hat{S} = \hat{s}^d$$

Study Design

- Search for nests
  - Encounter active nest: revisit at intervals until
    - Failure
    - Fledging
- Intervals $l = 1,\ldots,L$
- $n_{l.}$ = Number of intervals of duration *l* for which fate was recorded
  - $= n_{ls} + n_{lf}$ (n with success, n with failures)

Mayfield's Estimator

- MLE for case where all nests visited each interval (every day)

$$\hat{s} = \frac{n_s}{n_.}; \operatorname{var}(\hat{s}) = \frac{\hat{s}(1-\hat{s})}{n_.}$$

- $n_.$ = Number of intervals of duration 1 for which fate was recorded
  - $= n_s + n_f$ (n_s with successes, n_f with failures)

More General Approach

CA-A--A---A----A-----A---N

CA--------A—A---------A

CA----A----A----N

CA--A---A-----A-----A-----A

CA = discovered nest; A = active; N = not active

- $n_{l.}$ = Number of intervals of duration *l* for which fate was recorded
  - $= n_{ls} + n_{lf}$ (n_{ls} with success, n_{lf} with failures)

Example L=6

Interval length	$n_{l.}$	n_{ls}
1 (s)	81	79
2 (s^2)	30	29
3 (s^3)	61	61
4 (s^4)	37	36
5 (s^5)	11	10
6 (s^6)	21	19

$$\hat{s} = 0.989, (0.981, 0.997)$$

Estimation

- Binomial for each *l*, with survival constant but taken to appropriate power

$$f(n_{ls} \mid n_{l.}, s) = \prod_{l=1}^{L} \frac{n_{l.}!}{n_{ls}!\,n_{lf}!} (s^l)^{n_{ls}} (1-s^l)^{n_{lf}}$$

- MARK can be used to estimate *s*

Assumptions

- Rates constant
  - Accommodate variation through stratification
- Visits recorded
- Pr(*s*) not influenced by observer
- Pr(visit) independent of Pr(survival)

Untrastructural model for covariates

- Associate factors with success

$$s = \frac{e^{(\beta_0 + \sum_j \beta_j x_j)}}{1 + e^{(\beta_0 + \sum_j \beta_j x_j)}}$$

Nest Success Models: Extensions/Advances

- Pollock and Cornelius (1988)
  - Use age of nest when first encountered to estimate age (stage) specific survival of nests
- Natarjan and McCulloch (1999)
  - Random-effects model (heterogeneity)
- Dinsmore et al. (2002)
  - Age-specific survival to approximate heterogeneity
- Rotella et al. (2004)
  - Review of approaches
- Etterson et al. (2007)
  - Partioning risk (causes of failure)

Design Issues: Nest Success

- Can predict *n* of samples (nests) needed
- Tension between more nests and more visits
  - Fewer visits & more nests = increased precision
  - Fewer visits = less information on stage transitions and fledging
- Need to worry about time of fledging
  - If miss final days, confuse mortality for success
  - Possible positive bias in survival rate
  - Age nests, and time visits for first possible day of fledging; do not include exposure days after that date
  - More formal approaches: Stanley (2000), Stanley and Grubb (2004)

Radiotelemetry Studies

- Unlike nest studies, no natural endpoint
- Heisey and Fuller (1985) approach
  - Assumes that survival may vary among intervals L_i, i=1,…,k (over time)
  - x_i Number of "transmitter" days
  - d_i Number of deaths in interval

$$\hat{s}_i = \frac{(x_i - d_i)}{x_i} \qquad \hat{S} = \prod_{i=1}^{k} \hat{s}_i^{L_i}$$

  - Censuring can occur (varies number of transmitter days)

Source-Specific Mortality

- m_{ij} = Pr(animal alive during a day in interval i dies during the day from mortality source j)
- d_{ij} = number of deaths in interval i resulting from mortality source j

$$\hat{m}_{ij} = \frac{(d_{ij})}{x_i}$$

- Probability that animal dies as result of source j during interval i is

$$\hat{M}_{ij} = \hat{m}_{ij} + \hat{s}_{ij}\hat{m}_{ij} + \hat{s}^{2}_{ij}\hat{m}_{ij} + .. + \hat{s}^{L_i-1}_{ij}\hat{m}_{ij}$$

$$= [\hat{m}_{ij}/(1-\hat{s}_i)](1-\hat{s}^{L_i}_i)$$

Kaplan-Meier Nonparametric Survival Model

- Discrete hazard function h_j;
  Pr(die on day j | alive just prior to j)
  - d_j = # of animals that die at time j, for observed times at death, t_1, t_2,… t_j
  - r_j = # of animals at risk in just prior to t_j

$$\hat{h}_j = \frac{d_j}{r_j}$$

- Empirical estimate of $S(t)$;
  Pr(survive until at least t) $\hat{S}(t) = \prod_{j=1}^{t}(1-\hat{h}_t)$
- Product limit estimator

Kaplan-Meier Nonparametric Survival Model

- Pollock et al (1989) noted that model and estimator can accommodate
  - Addition of animals
  - Temporary censoring
  - Permanent censoring
  - modify r_j to subtract deaths and censoring and add new animals

Parametric and Nonparametric Survival Models

- Failure time analysis: huge field
  - Models of time until failure
  - Model instantaneous hazard rate h(t)
- Earlier models:
  - Fixed time periods, binomial models
- Parametric models (model survival function)
  - Proportional hazards, Weibull
- Kaplan-Meier Nonparametric Survival Model

Design Considerations I

- Capture *n* animals
  - How many? Use binomial model for sample allocation
  - "Staggered entry" is OK
- Postrelease adjustment period
  - Minimize possible behavioral and radio effects

Design Considerations II

- Periodically survey area, record fates (alive or dead), cause of death
  - Study area must be small enough to permit frequent surveys
  - Try to prevent censoring
  - Animals not encountered should be censored, and if later resighted should be considered as a new staggered entry*
  - Censoring must be random and independent of fate
    - Frequently violated in studies